Load Connected to the Motor by a Gear

• A gear is sometimes placed between the motor and the load with the aim of adjusting the loading on the motor.
• If the motor has to operate with maximum static error $\theta_{eM}$ then the motor has to operate with a static position error of $\theta_{eM} = N\theta_{eL}$.
• If the load torque is $T_L$ then the torque on the motor is $T_L/N$ (neglecting friction torque effects).
• For static operation, there is considerable advantage to using a high gear ratio to link the motor and load.
• Effects of gear friction and backlash degrade system performance. We assume an ideal gear train: negligible inertia, friction, and backlash.

• During dynamic operation

\[ T_L = J_L \frac{d^2 \theta_L}{dt^2} \]

\[ T_M = \frac{T_L}{N} = \frac{J_L}{N^2} \frac{d^2 \theta_M}{dt^2} \]

• The effective inertia at the motor is \( J_L/N^2 \). A high gear ratio would enable the motor to accelerate rapidly - BUT remember \( \theta_L = \theta_M/N \).
• **In summary:**
  
  – **High gear ratio**
    - Low reflected inertia
    - Fast acceleration
    - Short load step length
    - High motor speed (high stepping rate)
    - Use when load movement involves substantial periods of acceleration and deceleration
  
  – **Low gear ratio**
    - High reflected inertia
    - Slow acceleration
    - Long load step length
    - Low motor speed (low stepping rate)
Gear Train Relations:

\[ \theta_m \theta_m' = \frac{N_2}{N_1} \equiv N \]

\[ \frac{T_m}{T_m'} = \frac{N_1}{N_2} \equiv \frac{1}{N} \]

Motor Connected to Load by a Gear
Load Connected to the Motor by a Leadscrew

- Many linear loads are driven from a rotary stepping motor by a leadscrew, which may be an integral part of the motor.
- One revolution of the motor causes a load movement equal to the pitch $h$ of the screw:
  \[
  \frac{\theta}{2\pi} = \frac{x}{h}
  \]
- If the load is subject to a force $F$ then, neglecting leadscrew friction,
  \[
  F_x = T_L \theta
  \]
  \[
  T_L = \frac{F_x}{\theta} = \frac{Fh}{2\pi}
  \]
• The static position error for a system subject to a load force can be calculated as follows:
  – Calculate the effective load torque at the motor
  – From the motor static torque / rotor position error characteristic calculate the error in the motor’s rotational position
  – Translate the rotational error into a linear error
• If the load is to be accelerated:

\[
F = M \frac{d^2x}{dt^2}
\]

\[
T_L = \frac{Fh}{2\pi} = \frac{Mh}{2\pi} \frac{d^2x}{dt^2} = M \left( \frac{h}{2\pi} \right)^2 \frac{d^2\theta}{dt^2}
\]

\[
J_{eff} = M \left( \frac{h}{2\pi} \right)^2
\]
• In our analysis we assume an ideal leadscrew: negligible inertia, friction, and backlash.

• From a static point of view, a small screw pitch reduces the load torque at the motor.

• In the dynamic situation, there is a close parallel between the use of a small screw pitch and a high gear ratio. For small \( h \), \( J_{\text{eff}} \) is reduced and the motor can accelerate rapidly, but it must attain a high stepping rate to compensate for the small increments of linear movement produced by each motor step.
Motor Connected to Load by a Leadscrew