Mechatronic System Case Study: Rotary Inverted Pendulum Dynamic System Investigation

Dr. Kevin Craig
Greenheck Chair in Engineering Design & Professor of Mechanical Engineering
Marquette University
Innovation + Integration + Optimization

All Done Simultaneously from the Beginning of the Design Process!

Performance, reliability, low cost, robustness, efficiency, and sustainability are absolutely essential.
Mechatronics is the **synergistic integration** of physical systems, electronics, controls, and computers through the design process, from the very start of the design process, thus enabling complex decision making.

Integration is the key element in mechatronic **design** as complexity has been transferred from the mechanical domain to the electronic and computer software domains.

**Mechatronics is an evolutionary design development** that demands horizontal integration among the various engineering disciplines as well as vertical integration between design and manufacturing.

**Mechatronics is the best practice for synthesis** by engineers driven by the needs of industry and human beings.

---

Rotary Inverted Pendulum
Dynamic System Investigation

K. Craig 3
Mechatronic System Design Process

START HERE

System Design Concept
- Simplifying Assumptions
- Apply Laws of Nature
- Engineering Judgment

Concept Physical Model
- Identify Model Parameters
- Re-evaluate Physical Model Assumptions & Parameters

Concept Mathematical Model
- Solve Equations: Analytical & Numerical

Control System Design
- Design & Simulate
- Re-evaluate

Expected Closed-Loop System Response
- Past Experience & Experiments

Predicted Closed-Loop System Response
- Improve Control Design: Feedback, Feedforward, Observers, Filters

Is predicted response acceptable with respect to specifications?

Yes
- Improve System Design: Parameters and/or Configuration / Concept

No
- Build and Test Physical System
- Check that System Meets Specifications
- Evaluate, Iterate, and Improve As Needed

Solve Equations: Analytical & Numerical

Expected Component and Open-Loop System Response
- Past Experience & Experiments

Predicted Open-Loop System Response
- Yes
- Agreement?

No
- Build and Test Physical System
- Check that System Meets Specifications
- Evaluate, Iterate, and Improve As Needed

K. Craig  4

Rotary Inverted Pendulum
Dynamic System Investigation
The Mechatronic System Design Process provides an environment that is rich with numerical and graphical analysis and design tools that stimulate innovation, integration, communication, and collaboration within design teams. It aims to reduce the risk of not meeting the functional requirements by enabling early and continuous verification throughout the entire design workflow.

There are no free lunches in design; there are always tradeoffs. The best path to good design is to become aware of these tradeoffs, assess the effects of these tradeoffs through modeling and analysis, and then make an intelligent choice based on what you need.
In evaluating concepts, a modeling-and-analysis approach must replace any design-build-and-test approach, but this modeling is multidisciplinary and crosses domain boundaries.
Mechatronic System Case Study:
Rotary Inverted Pendulum
Dynamic System Investigation

Physical System
Physical System Components

- Two Links
  - Motor-driven horizontal link
  - Un-actuated vertical pendulum link
- Permanent-magnet, brushed DC motor actuator
- 24-volt, 5-amp, DC power supply
- Pulse-width-modulated servo-amplifier
- Two rotary incremental optical encoders
  - The resolution with quadrature decoding of each encoder is: 2000 cpr (horizontal link) and 5000 cpr (pendulum)
- One encoder measures pendulum angle
- One encoder measures horizontal link angle
- Velocity data is derived for each link from the encoder data

- Slip-ring assembly, mounted between the housing and the motor shaft, used to connect power to the pendulum optical encoder and read the signal from the three channels of the encoder
- Counter-weight on the horizontal arm
- Leveling screws on the housing base
- The command to the amplifier is provided by a MatLab-based real-time control system.
Physical Model: Simplifying Assumptions

- Links are rigid
- Two-degree-of-freedom system; generalized coordinates are:
  - $\theta$ - horizontal link angle
  - $\phi$ - pendulum arm angle
- Assume both Coulomb and viscous friction in the motor, in the slip-ring assembly, and at the pendulum revolute joint and perform tests to identify those parameters
- Dynamic response of the encoders is sufficiently fast that it can be considered instantaneous
- Dynamic response of the servo-amplifier is sufficiently fast that it can be considered instantaneous
• Motor operates in the torque mode with $V_{in}K_A = i$ and $T = K_Ti$ where:
  – $K_T$ is the motor torque constant (N-m/A)
  – $K_A$ is the amplifier constant (A/V)
  – $V_{in}$ is the command voltage (V)
  – $T$ is the electromagnetic torque applied to the motor rotor (N-m)
  – $i$ is the motor current (A)

• Motor is modeled in a lumped parameter way where:
  – $J$ is the rotor inertia (N-m-s²/rad = kg-m²)
  – $B_f$ is the viscous friction coefficient (N-m-s/rad)
  – $T_f$ is the Coulomb friction torque (N-m)
Rotary Inverted Pendulum
Dynamic System Investigation

**Reference Frames:**
R: ground xyz
R<sub>1</sub>: arm x<sub>1</sub>y<sub>1</sub>z<sub>1</sub>
R<sub>2</sub>: pendulum x<sub>2</sub>y<sub>2</sub>z<sub>2</sub>

\[
\begin{bmatrix}
\hat{i}_1 \\
\hat{j}_1 \\
\hat{k}_1
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{i} \\
\hat{j} \\
\hat{k}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\hat{i}_2 \\
\hat{j}_2 \\
\hat{k}_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
\hat{i}_1 \\
\hat{j}_1 \\
\hat{k}_1
\end{bmatrix}
\]

Top View

Arm Link 1

Pendulum Link 2

Front View

K. Craig 12
- **Angular Velocities of Links**

\[ R \vec{\omega}^{R_1} = \dot{\theta} \hat{k} = \dot{\theta} \hat{k}_1 \]

\[ R \vec{\omega}^{R_2} = \dot{\phi} \cos \theta \hat{i} + \dot{\phi} \sin \theta \hat{j} + \dot{\theta} \hat{k} \]

\[ = \dot{\phi} \hat{i}_1 + \dot{\theta} \hat{k}_1 \]

\[ = \dot{\phi} \hat{i}_2 + \dot{\theta} \sin \phi \hat{j}_2 + \dot{\theta} \cos \phi \hat{k}_2 \]

- **Velocities of CG’s of Links**
  - Point A is CG of Link 1
  - Point C is CG of Link 2

\[ R \vec{V}^A = (-\ell_{11} \dot{\theta} \sin \theta) \hat{i} + (\ell_{11} \dot{\theta} \cos \theta) \hat{j} \]

\[ R \vec{V}^C = (-\dot{\theta} \ell_1 \sin \theta - \dot{\theta} \ell_{21} \cos \phi \cos \theta + \dot{\phi} \ell_{21} \sin \phi \sin \theta) \hat{i} \]

\[ + (\dot{\theta} \ell_1 \cos \theta - \dot{\theta} \ell_{21} \cos \phi \sin \theta - \dot{\phi} \ell_{21} \sin \phi \cos \theta) \hat{j} \]

\[ + (\dot{\phi} \ell_{21} \cos \phi) \hat{k} \]
Rotary Inverted Pendulum
Dynamic System Investigation

\begin{align*}
\left( Rv^A \right)^2 &= \ell_{11}^2 \dot{\theta}^2 \\
\left( Rv^C \right)^2 &= \ell_{21}^2 \dot{\phi}^2 + \ell_{1}^2 \dot{\theta}^2 - 2\dot{\phi} \dot{\theta} \ell_{1} \ell_{21} \sin \phi + \dot{\theta}^2 \ell_{21}^2 \cos^2 \phi
\end{align*}

• **Definitions:**

\begin{align*}
\ell_{1} &= \text{length of link 1} = \ell_{11} + \ell_{12} \\
\ell_{11} &= \text{distance from pivot O to CG of link 1} \\
\ell_{12} &= \text{distance from CG of link 1 to end of link 1} \\
\ell_{2} &= \text{length of link 2} = \ell_{21} + \ell_{22} \\
\ell_{21} &= \text{distance from pivot B to CG of link 2} \\
\ell_{22} &= \text{distance from CG of link 2 to end of link 2}
\end{align*}
Lagrange’s Equations

- **Lagrange’s Equations**
  \[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \]

- **Generalized Coordinates**
  \[ q_1 = \theta \]
  \[ q_2 = \phi \]

- **Kinetic Energy** \( T \) of System
  \[
  T = \frac{1}{2} m_1 \left( R v^A \right)^2 + \frac{1}{2} I_{z1} \dot{\theta}^2 + \frac{1}{2} m_2 \left( R v^C \right)^2 + \\
  \frac{1}{2} \left[ I_{xz2} \dot{\phi}^2 + I_{yz2} (\sin^2 \phi) \dot{\theta}^2 + I_{zz2} (\cos^2 \phi) \dot{\theta}^2 \right] + I_{x2y2} \dot{\phi} \dot{\theta} \sin \phi
  \]
• **Potential Energy** $V$ of the System

$$V = -m_2 g l_{21} (1 - \sin \phi)$$

• **Generalized Forces**

$$Q_{\theta} = T - B_{\theta} \dot{\theta} - T_{f\theta} \text{sgn}(\dot{\theta})$$

$B_{\theta}$ = viscous damping constant $\theta$ joint

$T_{f\theta}$ = Coulomb friction constant $\theta$ joint

$$Q_{\phi} = -B_{\phi} \dot{\phi} - T_{f\phi} \text{sgn}(\dot{\phi})$$

$B_{\phi}$ = viscous damping constant $\phi$ joint

$T_{f\phi}$ = Coulomb friction constant $\phi$ joint

• **Equations of Motion**

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\theta}} - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = Q_{\theta}$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\phi}} - \frac{\partial T}{\partial \phi} + \frac{\partial V}{\partial \phi} = Q_{\phi}$$
Nonlinear Equations of Motion

\[
\begin{align*}
\left[ m_1 \ell_{11}^2 + I_{1z} + m_2 \ell_{21}^2 + m_2 \ell_{22}^2 \cos^2 \phi + I_{22} \cos^2 \phi + I_{2y} \sin^2 \phi \right] \ddot{\theta} + \\
\left[ I_x - m_2 \ell_{11} \ell_{12} \right] \sin \phi \ddot{\phi} + \left[ I_{x x} - m_2 \ell_{11} \ell_{12} \right] \cos \phi \phi^2 + \\
\left[ I_{y 2} - m_2 \ell_{21}^2 - I_{z 2} \right] (2 \cos \phi \sin \phi) \ddot{\phi} \dot{\theta} = T - \left[ B \dot{\theta} + T_{f \theta} \text{sgn} (\dot{\theta}) \right]
\end{align*}
\]

[1]

\[
\begin{align*}
\left[ m_2 \ell_{21}^2 + I_{2} \right] \ddot{\phi} + \left[ I_{2 \times 2 y} - m_2 \ell_{11} \ell_{21} \right] \sin \phi \ddot{\theta} + \\
\left[ m_{2 \ell_{21}^2 - I_{2 y} + I_{2 z} \right] (\cos \phi \sin \phi) \dot{\theta}^2 + m_2 g \ell_{21} \cos \phi = -\left[ B \phi \dot{\phi} + T_{f \phi} \text{sgn} (\dot{\phi}) \right]
\end{align*}
\]

[2]
Define: \( \alpha = \frac{\pi}{2} - \phi \)

\[
\begin{align*}
\left[ m_1 l_{11}^2 + I_{x_1} + m_2 l_{11}^2 + m_2 l_{21}^2 \sin^2 \alpha + I_{x_2} \sin^2 \alpha + I_{y_2} \cos^2 \alpha \right] \ddot{\theta} - \\
\left[ I_{x_{2y_2}} - m_2 l_{11} l_{21} \right] \cos \alpha \dddot{\theta} + \left[ I_{x_{2y_2}} - m_2 l_{11} l_{21} \right] \sin \alpha \dddot{\theta}^2 + \\
\left[ + I_{x_2} + m_2 l_{21}^2 - I_{y_2} \right] (2 \cos \alpha \sin \alpha) \dot{\alpha} \dot{\theta} = T - \left[ B_0 \dot{\theta} + T_f \text{sgn}(\dot{\theta}) \right]
\end{align*}
\]

[1A]

\[
\begin{align*}
- \left[ m_2 l_{21}^2 + I_{x_2} \right] \dddot{\theta} + \left[ I_{x_{2y_2}} - m_2 l_{11} l_{21} \right] \cos \alpha \dddot{\theta} + \\
\left[ m_2 l_{21}^2 - I_{y_2} + I_{x_2} \right] (\cos \alpha \sin \alpha) \dot{\theta}^2 + \\
m_2 g l_{21} \sin \alpha = \left[ B_\alpha \dot{\alpha} + T_{f_\alpha} \text{sgn}(\dot{\alpha}) \right]
\end{align*}
\]

[2A]
\[ \begin{align*}
\text{Linearization:} & \quad \theta = 0 \quad \alpha = 0 \\
\{ \begin{array}{l}
\left[ m_1 \ell_{11}^2 + I_{z1} + m_2 \ell_1^2 + I_{y2} \right] \ddot{\theta} - \left[ I_{x2y2} - m_2 \ell_1 \ell_{21} \right] \ddot{\alpha} = T - B_0 \dot{\theta} \\
- \left[ m_2 \ell_{21}^2 + I_{x2} \right] \ddot{\alpha} + \left[ I_{x2y2} - m_2 \ell_1 \ell_{21} \right] \dot{\theta} + m_2 g \ell_{21} \alpha = B_\alpha \dot{\alpha}
\end{array} \right. \\
\text{Definitions:} & \\
C_1 \dot{\theta} + C_2 \ddot{\alpha} = T - B_0 \dot{\theta} \quad [5] \\
C_3 \ddot{\alpha} + C_2 \dot{\theta} - C_4 \alpha = -B_\alpha \dot{\alpha} \quad [6] \\
\begin{align*}
C_1 &= m_1 \ell_{11}^2 + I_{z1} + m_2 \ell_1^2 + I_{y2} \\
C_2 &= m_2 \ell_1 \ell_{21} - I_{x2y2} \\
C_3 &= m_2 \ell_{21}^2 + I_{x2} \\
C_4 &= m_2 g \ell_{21}
\end{align*}
\end{align*} \]
Transfer Functions (neglect damping terms):

\[ \theta = \frac{C_3 s^2 - C_4}{T s^2 \left[ \left( C_1 C_3 - C_2^2 \right) s^2 - C_1 C_4 \right]} \]

\[ \alpha = \frac{-C_2 s^2}{T \left[ \left( C_1 C_3 - C_2^2 \right) s^2 - C_1 C_4 \right]} \]

State-Space Equations (neglect damping terms):

\[ \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{C_2 C_4}{C_1 C_3 - C_2^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{C_1 C_4}{C_1 C_3 - C_2^2} & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_3}{C_1 C_3 - C_2^2} \\ 0 \\ \frac{-C_2}{C_1 C_3 - C_2^2} \end{bmatrix} [T] \]
Model Parameter Identification

- Motor Parameters
- Masses of Links 1 and 2
- Location of CG’s of Links 1 and 2
- Moment of Inertia for Link 1: $\overline{I}_{z1}$
- Inertia Matrix for Link 2:

$$
\begin{bmatrix}
\overline{I}_{x2} & \overline{I}_{x2y2} & \overline{I}_{x2z2} \\
\overline{I}_{y2x2} & \overline{I}_{y2} & \overline{I}_{y2z2} \\
\overline{I}_{z2x2} & \overline{I}_{z2y2} & \overline{I}_{z2}
\end{bmatrix}
$$

- System Friction: Coulomb and Viscous
Motor Parameters: Given by Manufacturer

- $K_b = 0.0822$; back-emf constant (V-s/rad)
- $K_t = 0.0833$; torque constant (N-m/A)
- $R = 1.0435$; resistance (ohms)
- $L = 3.3E-3$; inductance (H)
- $B_m = 0$; viscous damping constant (N-m-s/rad)
- $T_f = 0.0124$; Coulomb friction (N-m)
- $J = 4.1E-5$; inertia (kg-m$^2$)
• **Link Masses (experimental)**
  - Link 1 (horizontal link): 0.911 kg
  - Link 2 (pendulum link): 0.133 kg

• **Location of CG’s for Links 1 and 2 (experimental)**
  - Link 1 (horizontal link): \( \bar{r} = -0.01 \text{ m} \)
  - Link 2 (pendulum link): \( \bar{r} = 0.188 \text{ m} \)
• Pendulum Link (Link 2): Friction and Inertia
  – Initially Assume Viscous Damping (Coefficient B)

\[
\begin{align*}
J_0 \ddot{\theta} + B \dot{\theta} + mg\bar{r} \sin \theta &= 0 \quad \sin \theta \approx \theta \\
J_0 \ddot{\theta} + B \dot{\theta} + mg\bar{r} \theta &= 0 \\
\ddot{\theta} + \frac{B}{J_0} \dot{\theta} + \frac{mg\bar{r}}{J_0} \theta &= 0 \\
2\zeta \omega_n &= \frac{B}{J_0} \quad \omega_n^2 = \frac{mg\bar{r}}{J_0} \quad \omega_n = 5.48 \text{ rad/s} \quad \bar{r} = 0.188 \text{ m} \quad m = 0.133 \text{ kg} \\
J_0 &= \frac{mg\bar{r}}{\omega_n^2} = 0.0082 \text{ kg-m}^2 \quad \bar{J} = J_0 - m\bar{r}^2 = 0.00347 \text{ kg-m}^2 \\
\delta &= \frac{1}{n} \ln \left( \frac{\theta_1}{\theta_{n+1}} \right) = 0.02 \quad \zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} = 0.003 \\
B &= 2\zeta \omega_n J_0 = 0.00027 \text{ N-m-s/rad}
\end{align*}
\]
Experimental Results: Free Oscillation of the Pendulum Link

Frequency of oscillation = 0.87 cycles/sec

Friction is a combination of viscous and Coulomb
Simulation of the Free Oscillation of the Pendulum Link
Simulation Results: Free Oscillation of the Pendulum Link

\[ B = 2.0 \times 10^{-4} \, \text{N-m rad/s} \]

\[ T_{f\alpha} = 1.8 \times 10^{-4} \, \text{N-m} \]
– Horizontal Link: Inertia

- Test horizontal link on a frictionless pivot
- Test Result: 10 cycles in 27.4 seconds
- Computed inertia does not include motor and slip-ring inertia; however, these are small compared to the computed inertia of the horizontal link and will be neglected

\[ J_o \ddot{\theta} + mg\bar{r} \sin \theta = 0 \quad \sin \theta \approx \theta \]

\[ J_o \ddot{\theta} + mg\bar{r} \theta = 0 \]

\[ \omega_n^2 = \frac{mg\bar{r}}{J_o} \quad \omega_n = 2.29 \text{ rad/s} \quad \bar{r} = .01 \text{ m} \quad m = 0.911 \text{ kg} \]

\[ J_o = \frac{mg\bar{r}}{\omega_n^2} = 0.017 \text{ kg-m}^2 \quad \bar{J} = J_o - m\bar{r}^2 = 0.0169 \text{ kg-m}^2 \]
– Horizontal Link Motor and Slip-Ring Friction: Coulomb Friction

\[
J_o \ddot{\theta} + mg \bar{r} \sin \theta + T_f = 0 \quad \sin \theta \approx \theta
\]

\[
J_o \ddot{\theta} + mg \bar{r} \theta + T_f = 0
\]

\[
\theta_n - \theta_{n-1} = - \frac{4T_f}{mg \bar{r}}
\]

\[
T_f = \frac{mg \bar{r}}{4} (\theta_{n-1} - \theta_n)
\]

\[
J_o = (0.017 + 0.0036 + 0.0017) = 0.0223 \text{ kg-m}^2
\]

\[
m = (0.911 + 0.3415 + 0.3415) \text{ kg} = 1.594 \text{ kg}
\]

\[
\theta_{n-1} - \theta_n = 0.219 \text{ rad}
\]

\[
\bar{r} = \frac{(0.911)(0.01) + (0.342)(0.102) + (0.342)(0.070)}{(0.911 + 0.3415 + 0.3415)} = 0.043 \text{ m}
\]

\[
T_f = 0.0368 \text{ N-m}
\]

System was modified, i.e., masses were added to produce oscillations. Does this change friction value determined? How can we check this?
Experimental Results: Free Oscillation of the Horizontal Link

θ₁ = 0.637
θ₂ = 0.409
θ₃ = 0.199
Simulation of Modified Horizontal Link Free Oscillation
Simulation Results: Free Oscillation of the Modified Horizontal Link
• Pendulum Inertia Matrix: Computational Results

\[
\begin{bmatrix}
\bar{I}_{2x_2} & \bar{I}_{2x_2y_2} & \bar{I}_{2x_2z_2} \\
\bar{I}_{2y_2x_2} & \bar{I}_{2y_2} & \bar{I}_{2y_2z_2} \\
\bar{I}_{2z_2x_2} & \bar{I}_{2z_2y_2} & \bar{I}_{2z_2}
\end{bmatrix}
= \\
\begin{bmatrix}
3.5374 \times 10^{-3} & -6.5383 \times 10^{-5} & 0 \\
-6.5383 \times 10^{-5} & 2.8457 \times 10^{-5} & 0 \\
0 & 0 & 3.5430 \times 10^{-3}
\end{bmatrix} \text{kg-m}^2
\]

• Experimental Result \( \bar{I}_{2x_2} = 0.00347 \text{ kg-m}^2 \)
Horizontal Arm Parameters

- \( g = 9.81 \text{ m/s}^2 \)
- \( L_{11} = -0.01 \text{ m} \)
- \( L_1 = 0.21 \text{ m} \)
- \( M_1 = 0.911 \text{ kg} \)
- \( I_{\text{bar}_1 \ z1} = 0.0169 \text{ kg-m}^2 \)
- \( T_{f \_\text{theta}} = 0.0368 \text{ N-m} \)
- \( B_{\text{theta}} = 0 \text{ N-m-s/rad} \)

Motor Parameters

- \( K_b = 0.0822 \text{ V-s/rad} \)
- \( K_t = 0.0833 \text{ N-m/A} \)
- \( R_{\text{motor}} = 1.0435 \text{ ohms} \)
- \( L = 3.3 \times 10^{-3} \text{ H} \)
- \( B_{\text{motor}} = 0 \text{ N-m-s/rad} \)
- \( T_{f \_\text{motor}} = 0.0124 \text{ N-m} \)
- \( J = 4.1 \times 10^{-5} \text{ kg-m}^2 \)

Pendulum Parameters

- \( M_2 = 0.1326 \text{ kg} \)
- \( L_{21} = 0.1885 \text{ m} \)
- \( I_{\text{bar}_2 \ y2} = 2.8457 \times 10^{-5} \text{ kg-m}^2 \)
- \( I_{\text{bar}_2 \ x2} = 3.5374 \times 10^{-3} \text{ kg-m}^2 \)
- \( I_{\text{bar}_2 \ z2} = 3.5430 \times 10^{-3} \text{ kg-m}^2 \)
- \( I_{\text{bar}_2 \ x2y2} = -6.5383 \times 10^{-5} \text{ kg-m}^2 \)
- \( T_{f \_\phi} = 1.8 \times 10^{-4} \text{ N-m} \)
- \( B_{\phi} = 2.0 \times 10^{-4} \text{ N-m-s/rad} \)
Nonlinear Equations of Motion: Rotary Inverted Pendulum
Experimental Results: Total System Response - Link 1
Simulation Results: Total System Response - Link 1
Simulation vs. Experimental Results: Total System Response - Link 1

Experimental

Simulation
Experimental Results: Total System Response - Link 2
Rotary Inverted Pendulum Dynamic System Investigation

Simulation Results: Total System Response - Link 2

- Graph showing the simulation results for total system response of Link 2 over time.

- X-axis: Time (sec)
- Y-axis: phi (rad)
- Time range: 0 to 60 sec
- phi (rad) range: -5 to 2
Simulation vs. Experimental Results: Total System Response - Link 2

- Experimental
- Simulation

Time (sec):
- 0
- 10
- 20
- 30
- 40
- 50
- 60

Alpha (rad):
- 0
- 1
- 2
- 3
- 4
- 5
- 6

Rotary Inverted Pendulum
Dynamic System Investigation
K. Craig  41
Rotary Inverted Pendulum: Swing-Up Control and Balancing Control

MatLab / Simulink Control Block Diagram

Rotary Inverted Pendulum
Dynamic System Investigation

K. Craig 42
• **Balancing Controllers**
  – Full-State Feedback Regulator
  – Classical Control Design

• **Swing-Up Controller**
  – Calculates the total system energy based on the kinetic energy of both links and the potential energy of the pendulum.
  – The calculated total system energy is compared to a defined quantity of energy when the pendulum is balanced (i.e., zero energy when balanced).
  – The difference between the desired energy and the actual energy is multiplied by an “aggressivity” gain and applied to the motor.
The objective of the swing-up control exercise is to move the system from the stable equilibrium position to the unstable equilibrium position.

Energy must be added to the system to achieve this swing-up action.

The manipulated input to achieve this is given by the control law:

\[ V = K_A (E - E_0) \text{sgn} \left( \frac{da}{dt} \cos \alpha \right) \]

The first two terms in the above control law are the "aggressivity" gain and the difference between actual and desired system energy. These two terms provide the magnitude of energy that has to be added to the system at any given time.
– The "aggressivity" gain determines what proportion of the available input will be used to increase or decrease the system energy. This gain could be the difference in swinging the pendulum up in 3 or 10 oscillations.

– The second half of the energy swing up equation determines the direction the input should be applied to increase the energy of the system. The velocity term causes the input to change directions when the pendulum stops and begins to swing in the opposite direction. The cosine term is negative when the pendulum is below horizontal and positive above horizontal. This helps the driven link to get under the pendulum and catch it.
- By controlling on energy feedback, the system automatically stops inputting excess energy and allows the system to coast to a balanced position. When the remaining potential energy required is equal to the kinetic energy, the feedback will become very small and the pendulum will coast to vertical position.

- By setting the desired energy to a value greater than zero, unmodeled energy dissipation effects can be overcome as the pendulum is approaching its balanced point. If this is too much, the pendulum will overshoot and the driven link will not be able to catch it.
The switching between the controllers has a deadband of 5°. When the pendulum is within ± 25° of vertical, the swing up controller will turn off. If the pendulum coasts to within ± 20° of vertical, the balance controller will be activated and the driven link will attempt to catch the pendulum. If the balance controller is not successful, the pendulum will fall and the swing up algorithm will automatically engage.
Control Selection: Swing-up vs. Balance

1. Pendulum Angle Normalized
2. Swing-Up Control
3. Balancing Control

- Limit Switch + or - 25 degrees
- Limit Switch + or - 20 degrees

Control Torque

0.43633
-0.43632

0.34907
-0.34907
Simulation Results

State-Space Control vs. Classical Control

- State-Space Control
- Classical Control
Simulation Results

Simulation Comparison: State-Space Control vs. Classical Control

State-Space Control

Classical Control

alpha_n (rad)

0 1 2 3 4

1 2 3 4

0 -1 -2 -3 -4

time (sec)

0 1 2 3 4 5 6 7 8 9 10

Rotary Inverted Pendulum
Dynamic System Investigation

K. Craig 50