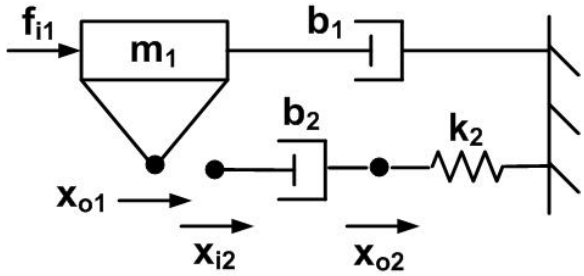
Impedance – An Essential Multidisciplinary Concept

Its Great Utility Has Been Neglected in Engineering Education and Practice

The Jeopardy category is Essential Engineering Concepts. The answer is: electrical engineers apply it all the time, while mechanical engineers most likely have not heard about it, but its utility in understanding and predicting how multidisciplinary systems perform when they are interconnected is unquestioned. The question is: what is impedance?



The diagram shows two mechanical subsystems, each a simple 1st-order system, which are to be connected. The first system might represent a machine tool slide being positioned by a motor providing the force f_{i1}; the second system could represent a velocity-measuring device we wish to attach to the slide to measure its speed.

There are three ways to correctly predict the dynamic behavior of the interconnected subsystems. The one commonly-used incorrect way is to derive each system transfer function separately and multiply them together, observing that $x_{o1} = x_{i2}$ when the systems are connected. This completely ignores that fact that the second subsystem loads, i.e., draws energy from, the first subsystem. Yet, this is typically how it is presented in controls courses where transfer functions connected in series in a block diagram are multiplied together. Rarely is there any mention of loading. It should not be a surprise that the actual interconnected systems do not behave as predicted.

The first correct approach is to consider the system as a whole, with the subsystems connected, and analyze it from scratch. We find the following correct result:

$$\frac{x_{o2}}{f_{i1}} = \frac{\dfrac{b_2}{(b_1+b_2)k_2}}{\dfrac{m_1 b_2}{(b_1+b_2)k_2}s^2 + \dfrac{m_1 k_2 + b_1 b_2}{(b_1+b_2)k_2}s + 1}$$

Each subsystem here has an in port and an out port, and at each port there are two variables, force and velocity, the product of which is instantaneous power. The second approach to completely describe the dynamic behavior of this system is to derive a 2 × 2 transfer-function matrix showing the relationship among these four variables. The result of such an analysis is:

$$\begin{bmatrix} f_{i1} \\ x_{i1} \end{bmatrix} = \begin{bmatrix} 1 & m_1 s^2 + b_1 s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_{o1} \\ x_{o1} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} f_{i2} \\ x_{i2} \end{bmatrix} = \begin{bmatrix} 1 & k_2 \\ \dfrac{1}{b_2 s} & \dfrac{b_2 s + k_2}{b_2 s} \end{bmatrix} \begin{bmatrix} f_{o2} \\ x_{o2} \end{bmatrix}$$

Recognizing that $f_{o1} = f_{i2}$ and $x_{o1} = x_{i2}$ when the systems are connected, these two matrices can be multiplied together to predict the correct complete system behavior.

These two approaches work well when one has a mathematical model for each subsystem, as the model-based design approach of mechatronics strives for. But sometimes, for a variety of reasons, all one has is the actual hardware without any models available; the subsystems must still be connected resulting in a well-performing complete system. How can an engineer ensure that this will happen without the benefit of modeling? The answer is the impedance concept, where impedance is generally defined as the ratio of the two variables whose product is power. When two systems are connected, the upstream ideal (unloaded) subsystem transfer function must be modified due to the loading effect caused by the downstream subsystem connection. That modification is to multiply the ideal upstream transfer function by the following term to produce what is called the loaded transfer function. Z_{o1} is the output impedance of subsystem 1 and Z_{i2} is the input impedance of subsystem 2.

$$\left[\frac{1}{1+\frac{Z_{o1}(s)}{Z_{i2}(s)}}\right] \quad \text{where} \quad Z_{o1}(s)=\left.\frac{x_{o1}}{f_{o1}}\right|_{f_{i1}=0} \quad \text{and} \quad Z_{i2}(s)=\left.\frac{x_{i2}}{f_{i2}}\right|_{f_{o2}=0}$$

Clearly, if $Z_{o1} << Z_{i2}$ over some frequency range of interest, then loading effects are negligible. This approach gives the exact result, as do the previous two approaches. So why is this approach so important? The subsystem ideal (unloaded) transfer functions, upstream and downstream, along with Z_{o1} and Z_{i2}, can be measured experimentally. Once they are measured, the results can be used to properly predict the behavior of the assembled system. Of course, this approach can also be used with mathematical models to predict the correct behavior.

You win! The correct question is “What is Impedance?” and it is not just for EEs!

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