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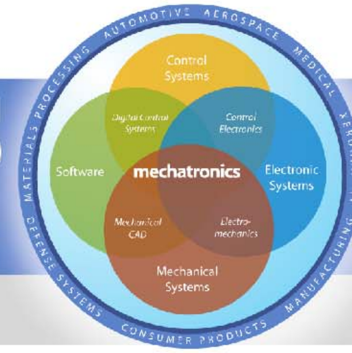

Inertia Mismatch: Fact or Fiction?

Is this often-quoted concept an excuse for inadequate system modeling?

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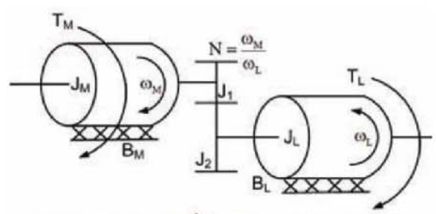
SERVO SYSTEMS may be direct-drive or geared systems. For geared systems, the choice of motor also involves a choice of gear ratio. **Figure 1** shows a motor connected to a load through an ideal (i.e., no friction, no backlash, and no compliance) gear train with gear ratio N. The equation of motion for this one-degree-of-freedom system is shown in terms of the load angular velocity, ω_L. The inertia ratio is the ratio of the load inertia to the motor inertia, J_L/J_M. What is the optimum value of this ratio for a particular value of N? If we ignore the motor and load damping, B_M and B_L, respectively, and the load torque, T_L, the inertia ratio that maximizes the power transferred from motor to load is N^2, i.e., the reflected motor inertia equals the load inertia. When friction and load torques and system compliance (e.g., coupling or timing-belt compliance) are significant, the selection of an optimum inertia ratio is less straightforward. **Figure 2** shows this more general case.

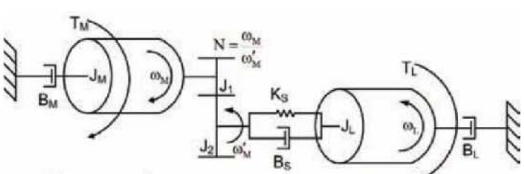
Why do deviations from the so-called ideal inertia ratio of N^2, called *inertia mismatch*, particularly in compliantly coupled systems, often cause servo-system stability problems? This is a *system* question and to answer this question we must examine the frequency response plot for a compliantly coupled motor and load, as shown in **Figure 3** ($N = 1$, $B_M = 0$, $B_L = 0$). The anti-resonance frequency ω_{AR} always occurs before the resonance frequency ω_R. At a low J_L/J_M ratio, the resonance and anti-resonance frequencies are close to each other at a high frequency. As J_L/J_M increases, both the anti-resonance and resonance frequency decrease, with the anti-resonance frequency decreasing at a faster rate. For a given J_L, to increase the resonance frequency, either increase the shaft stiffness, K_S, or decrease the motor inertia. As K_S increases, both ω_R and ω_{AR} increase. The smaller the inertia ratio, the less compliance will affect the system.

All mechanical systems have compliance, and the choice of the inertia ratio and transmission ratio (e.g., gear, belt, or lead screw) and the design of the feedback control system are done based on a system analysis and not by trusting poorly defined rules of thumb. **DN**



$$\left[N^2(J_M+J_1)+(J_L+J_2)\right]\frac{d\omega_L}{dt}+\left[N^2B_M+B_L\right]\omega_L = NT_M - T_L$$

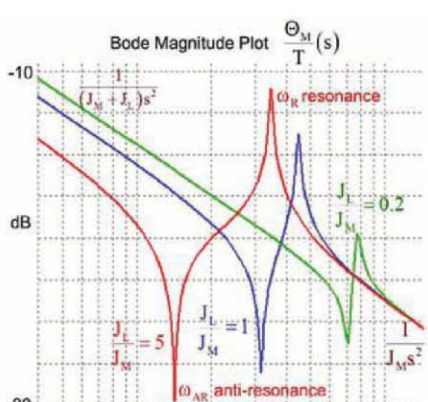
Figure 1: Motor connected to load by an ideal gear train.



$$\left[J_M + J_1 + J_2\left(\frac{1}{N}\right)^2\right]\frac{d\omega_M}{dt} + B_M\omega_M + B_S\left(\frac{1}{N}\right)\left(\frac{\omega_M}{N} - \omega_L\right) + K_S\left(\frac{1}{N}\right)\left(\frac{\theta_M}{N} - \theta_L\right) = T_M$$

$$J_L\frac{d\omega_L}{dt} + B_L\omega_L - B_S\left(\frac{\omega_M}{N} - \omega_L\right) - K_S\left(\frac{\theta_M}{N} - \theta_L\right) = -T_L$$

Figure 2: Motor connected to load by an ideal gear train with compliance.



$$\frac{\Theta_M}{T}(s) = \left[\frac{1}{(J_M + J_L)s^2}\right]\frac{J_L s^2 + B_S s + K_S}{\frac{J_L J_M}{J_L + J_M}s^2 + B_S s + K_S}$$

$$\omega_R = \sqrt{\frac{K_S(J_M + J_L)}{J_M J_L}} \qquad \omega_{AR} = \sqrt{\frac{K_S}{J_L}}$$

$J_M = 0.002$ kg-m^2
$K_S = 200$ N-m/rad
$B_S = 0.01$ N-m-s/rad

Figure 3: Frequency response of a compliantly coupled motor-load (N=1).