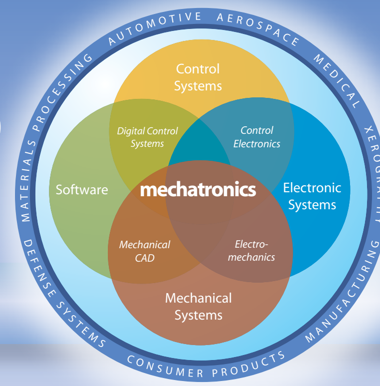
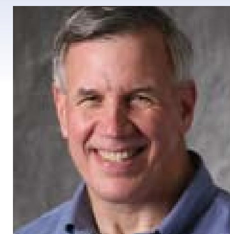


Stability — The Ultimate Limit to Performance



BY KEVIN CRAIG

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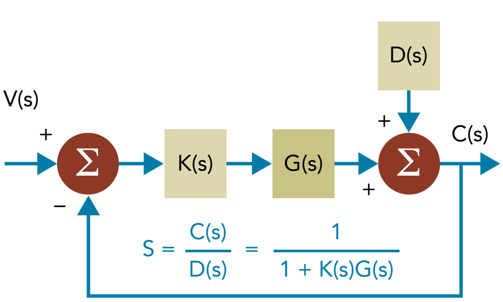
We all experience with amazement every day the power of feedback control to improve the performance of a tremendous range of technological systems. Feedback control is at the heart of every modern mechatronic system and simultaneous optimization of the control with the physical system and its sensors and actuators is paramount. Some systems cannot even operate without feedback control, which makes it an enabling technology. And, of course, feedback is used to regulate nearly every system in the human body and is constantly at work in ecological systems. What makes all this most challenging is that changes cannot be effected instantaneously in a dynamical system and a correct control decision applied at the wrong time could result in catastrophe, as can loss of control due to actuator saturation. As engineering systems are becoming more complex, many are becoming more dangerous. Control systems must be safe and robust, and guaranteed to be so, before any thoughts of performance are considered.

This brings me to the topic of this article — stability, the ultimate limit to performance. It should not be surprising that robustness achieved through feedback control is subject to limits. There is a robustness trade-off present in all feedback systems. An understanding of fundamental limitations, in practical, physical terms rather than abstract, mathematical terms, is an essential element in all engineering. Now, more than ever, the practical, physical consequences of control must be respected and the underlying principles thoroughly understood. As presented by Gunter Stein in his 1989 IEEE Bode Lecture (*IEEE Control Systems Magazine*, August 2003 — required reading for all control engineers), Hendrik Bode observed that there is a fundamental limitation on the achievable sensitivity function S (sensitivity to disturbances and modeling errors) for a feedback system (see figure, below). The log of the magnitude of the sensitivity function of a LTI (linear, time-invariant), SISO (single-input, single-output) feedback system, integrated over frequency, is conserved under the action of feedback — it is zero for stable plant/compensator pairs and is some fixed positive value for unstable ones.



Sensitivity Transfer Function.

Sensitivity improvements in one frequency range must be paid for with sensitivity deteriorations in another frequency range and the price is higher if the plant is open-loop unstable. However, physical systems do not exhibit good frequency response fidelity beyond a certain bandwidth. This is due to uncertain or unmodeled dynamics in the plant, to digital control implementations, to power limits and to nonlinearities, for example. So the integration is performed over a finite frequency range, a constraint imposed by the physical hardware we use in the control loop. All the action of the feedback design and the sensitivity improvements, as well as the sensitivity deteriorations, must occur within a finite frequency range. So reducing the sensitivity of a system to disturbances at one range of frequencies by feedback control will amplify transients and oscillations at other frequencies.

In his lecture, Gunter Stein applied Bode's conservation law to a well-known example of instability — the stick-balancing problem. A long rigid stick is easy to balance, but as it becomes shorter, balancing it becomes more difficult. Handicaps to human control, including reaction time, neuromuscular lags and limb inertias, limit compensator frequency range. The control strategy is to keep the sensitivity as small as possible over that range. However, a dramatic increase in sensitivity occurs as the stick becomes shorter and even minor imperfections in the implementation will cause instability. This is the reason humans have trouble balancing short sticks.

Gunter Stein felt every control engineer, and therefore every mechatronics' engineer, should understand this most important observation by Bode that has been neglected in controls' education. I could not agree more.

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