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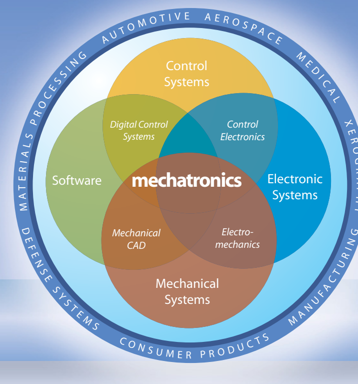


# MECHATRONICS IN DESIGN

FRESH IDEAS ON INTEGRATING MECHANICAL SYSTEMS, ELECTRONICS, CONTROL SYSTEMS AND SOFTWARE IN DESIGN

# System Motion Fundamentals

## How tossing stuff into the air helps us understand moments of inertia and principal axes which are essential for design

BY KEVIN CRAIG

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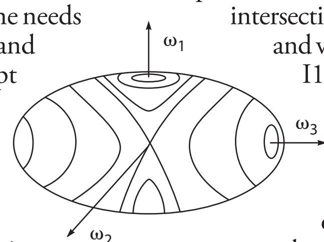
Take any book and wrap a few rubber bands around it. Toss the book in the air three times, each time giving it a pure rotation, as best you can, about one of the three axes perpendicular to its sides. What do you observe? This simple experiment demonstrates fundamentals essential to the design of rotating machines, space satellites and much more.

The motion of any system depends on the forces acting on it and its constitution, i.e., the manner in which its mass is distributed, usually in response to strength, weight, space and stiffness requirements. To predict dynamic behavior, all one needs to know are the mass, location of the mass center and six quantities called the inertia scalars. The concept of mass center is well known and its location is used to determine the translational motion of a body. But inertia scalars are not well understood. At any point in a body, one can determine six independent quantities called the three mass moments of inertia and the three products of inertia. Together they quantify how mass is distributed with respect to three mutually perpendicular axes fixed in the body at that point. The mass moments of inertia quantify the resistance of the body to angular acceleration about each axis, and the products of inertia quantify the symmetry of the mass distribution with respect to each plane. In addition, there is always a particular orientation of those axes such that the products of inertia are all zero. The remaining three quantities are called the principal mass moments of inertia and they play an important role in dynamic analysis, as we shall see.

Returning to our tossed book, the only force acting on the book is gravity and that force goes through the mass center. The book then is moment-free, spinning freely in space. So here we will neglect any translation of the book and just consider its rotation. Since the book is moment-free, the magnitude of its angular momentum vector, H, must be constant (conserved), and since we are neglecting translation, its rotational kinetic energy, T, must be constant (conserved). Plotting constancy of T and H using the absolute angular velocities ω1, ω2 and ω3 as ordinates gives two ellipsoids.

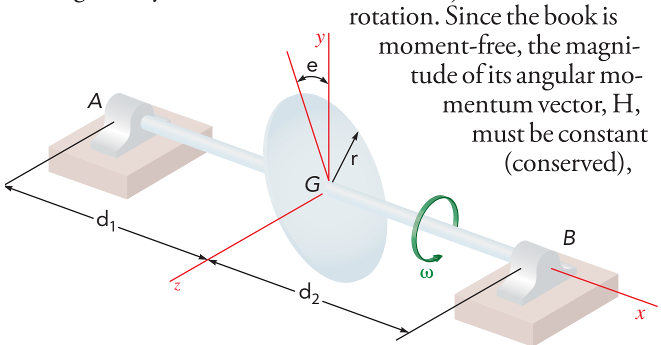



The only allowable spinning states are at the intersections of these two ellipsoids. The lines on the figure are the intersections for a fixed value of T and various values of H, where I1 > I2 > I3. The three intersections are circles at the greatest and least axes and a saddle at the intermediate axis. This indicates that rotation about the axes with the greatest and least moments of inertia is stable to small oscillations, while rotation with respect to the intermediate axis is unstable to small oscillations. This is just what you observed in the book experiment.

Another way to arrive at the same conclusion is by considering Euler's Equations for this situation, where the 1, 2, 3 axes are body-fixed principal axes through the mass center.

If the body is given a constant spin rate, Ω, exactly about any one if its principal axes, it will continue to spin about that axis. But what happens if that motion is perturbed by an angular velocity ωp? Let's assume ω1 = Ω + ωp. Analysis of Euler's Equations with linearization shows the resulting equation. If the coefficient of ω2 is negative, the solution for ω2 grows with time. This happens if the 2-axis is the intermediate principal axis.

Let's bring the topic of principal axes to everyday practice. Modern machines have high-speed rotors fastened to shafts, as shown in the figure. If the principal axis of the mounted object, here a homogeneous solid disk, does not coincide with the axis of the shaft, making the system dynamically balanced, as indicated by the angle ω = 0, then dynamic bearing reactions result that could lead to premature bearing failure.




Understanding fundamentals and being able to apply them to engineering applications is the hallmark of a complete engineer.

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